高等数学常用辅助公式

一、三角类

1. 和差化积与积化和差

$$\sin a + \sin b = 2\sin\frac{a+b}{2}\cos\frac{a-b}{2}$$

$$\sin a - \sin b = 2\cos\frac{a+b}{2}\sin\frac{a-b}{2}$$

$$\cos a + \cos b = 2\cos\frac{a+b}{2}\cos\frac{a-b}{2}$$

$$\cos a - \cos b = -2\sin\frac{a+b}{2}\sin\frac{a-b}{2}$$

$$\sin a\cos b = \frac{1}{2}[\sin(a+b)+\sin(a-b)]$$

$$\cos a\sin b = \frac{1}{2}[\sin(a+b)-\sin(a-b)]$$

$$\cos a\cos b = \frac{1}{2}[\cos(a+b)+\cos(a-b)]$$

$$\sin a\sin b = -\frac{1}{2}[\cos(a+b)-\cos(a-b)]$$

2. 二倍角

$$1-\cos x = 2\sin^2\frac{x}{2}$$

$$\cos 2x = 2\cos^2 x - 1 = 1 - 2\sin^2\frac{x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

3. 余切、正割、余割。

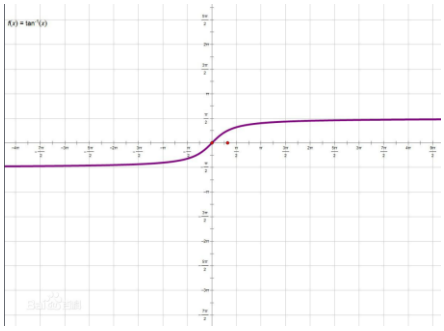
$$\begin{cases}\tan\alpha\cot\alpha = 1\\ \sin\alpha\csc\alpha = 1\\ \cos\alpha\sec\alpha = 1\end{cases}$$

$$\cot(x+\frac{\pi}{2}) = -\tan x$$

$$\csc x - \cot x = \tan x$$

☆ $\sec^2 x = \tan^2 x + 1$ ； $\csc^2 x = \cot^2 x + 1$

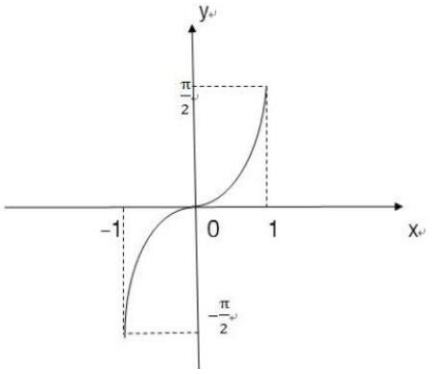
4. 反三角函数



$y=\arctan x$

定义域 $D_x=R$

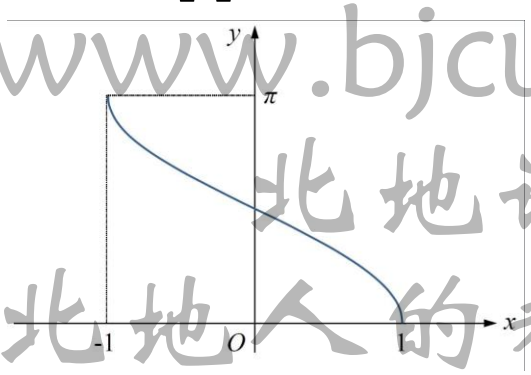
值域 $R_f=(-\frac{\pi}{2},\frac{\pi}{2})$



$y=\arcsin x$

$D_x=[-1,1]$

$R_f=[-\frac{\pi}{2},\frac{\pi}{2}]$



$y=\arccos x$

$D_x=[-1,1]$

$R_f=[0,\pi]$

二、工具

（1）. 基本公式

1. $\int k\,dx=kx+c$

2. ① $\int x^a dx=\frac{1}{a+1}x^{a+1}+c(a\neq -1)$

② $\int\frac{1}{x}dx=\ln|x|+c$

3. ① $\int a^x dx = \frac{a^x}{\ln a} + c$

② $\int e^x dx = e^x + c$

4. ① $\int \sin x dx = -\cos x + c$

② $\int \cos x dx = \sin x + c$

③ $\int \tan x dx = -\ln|\cos x| + c$

④ $\int \cot x dx = \ln|\sin x| + c$

⑤ $\int \sec x dx = \ln|\sec x + \tan x| + c$

⑥ $\int \csc x dx = \ln|\csc x - \cot x| + c$

⑦ $\int \sec^2 x dx = \tan x + c$

⑧ $\int \csc^2 dx = -\cot x + c$

⑨ $\int \sec x \tan x dx = \sec x + c$

⑩ $\int \csc x \cot x dx = -\csc x + c$

5. 平方和、平方差

① $\int \frac{dx}{\sqrt{1-x^2}} = \arcsin x + c$

② $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + c$

③ $\int \frac{dx}{1+x^2} = \arctan x + c$

④ $\int \frac{dx}{a^2+x^2} = \frac{1}{a} \arctan \frac{x}{a} + c$

⑤ $\int \frac{dx}{\sqrt{x^2+a^2}} = \ln(x + \sqrt{x^2+a^2}) + c$

⑥ $\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|x + \sqrt{x^2-a^2}\right| + c$

⑦ $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + c$

⑧ $\int \sqrt{a^2-x^2}\,dx = \frac{a^2}{2}\arcsin\frac{x}{a} + \frac{x}{2}\sqrt{a^2-x^2} + c$

三、基本不等式

$$\sqrt{\frac{a^2+b^2}{2}} \geq \frac{a+b}{2} \geq \sqrt{ab} \geq \frac{2}{\frac{1}{a}+\frac{1}{b}}$$

常用： $a+b \geq 2\sqrt{ab}$ ； $a^2+b^2 \geq 2ab$

$$a^2+b^2 \geq \frac{(a+b)^2}{2} \text{ ; } ab \leq (\frac{a+b}{2})^2$$

四、常见的麦克劳林级数

1. $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} (-\infty < x < +\infty)$

2. $\sin x = \sum_{0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} (-\infty < x < +\infty)$

3. $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n} (-\infty < x < +\infty)$

4. $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n (-1 < x < 1)$

5. $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n (-1 < x < 1)$

6. $\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^n (-1 < x \leq 1)$

   记：$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\ldots = \ln 2$

7. $-\ln(1-x) = \sum_{n=1}^{\infty} \frac{x^n}{n} (-1 \leq x < 1)$

五、基本高阶求导公式

1. $(\sin x)^n = \sin(x + \frac{n\pi}{2})$ $\qquad (\sin kx)^n = k^n \sin(kx + \frac{n\pi}{2})$

2. $(\cos x)^n = \cos(x + \frac{n\pi}{2})$

3. $(\frac{1}{ax+b})^n = \frac{(-1)^n n! a^n}{(ax+b)^{n+1}}$

六、leibniz 公式

$(uv)' = u'v + uv'$

$(uv)'' = (u'v + uv')' = u''v + u'v' + u'v' + uv'' = u''v + 2u'v' + uv'' = C_2^0 u''v + C_2^1 u'v' + C_2^2 uv''$